Measurement of Sports Surface Resilience II
- Energy Restitution

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Preamble

The springiness of a sports surface affects to some degree the performance of a player or athlete. The softer the track, the more is the deformation in the surface produced by the athlete's foot and hence loss of energy due to visco-elastic or friction effects, so there would seem to be an advantage in running on hard surfaces. Hard tracks, however, particularly non-resilient surfaces such as concrete, may lead to foot and leg injury after prolonged exposure. And, more recent results by Ferris et al (1998) have indicated that athletes may run faster on resilient surfaces. "A compliant elastic surface will store and return energy with each step, reducing the mechanical work performed by the runner's muscles." Their calculations suggest about 9 % of the energy of running would be stored and returned on a track of stiffness 195 kN/m.
In terms of energy dissipation within the surface these same authors have also calculated that it is about 2 - 4 % of the energy (vertical) of the athlete's centre of mass (or a 20 - 40 % energy loss in the surface). These figures apply to the solid rubber running tracks used in their investigations.

In Football the new (Third Generation) synthetic surfaces generally use a long pile tufted carpet filled with rubber crumbs and sand, and often supported by a rubber shock pad. The loose rubber and sand might be expected to be energy dissipative and for players running on such surfaces this may lead to a greater fatigue than might occur on the traditional natural turf surfaces. Thus there is an interest in measuring the energy dissipation that occurs during the interaction of the player with the surface.

An obvious first step in the examination of this phenomenon would be biomechanical studies of players performing on synthetic and on natural Football turfs. This is already in preparation under the auspices of UEFA, but there is also a need to develop testing methods that might characterise energy dissipating properties of the surfaces.
The most likely approach to achieving this is by using one of surface testing machines that simulate the dynamics of foot contact during a player's interaction with the surface. There are currently in use several different machines for the measurement of resilience of sports surfaces, which in their designs are attempts to simulate athlete interactions - the Berlin Athlete, the Stuttgart Athlete, Poitier, Clegg and HIC testers.
However most depart markedly from simulating the required forces and interaction times.

The biomechanics of running on surfaces have been intensively studied over the past decades, most notably by McMahon and Greene (1978) and more recently by Ferris et al (1998). They have proposed a mechanical model (spring mass) to represent the actions of the athlete, with parameters:

Effective vertical stiffness	30 kN/m on contact area of the athlete's sole
Mass (as for athlete)	60 kg
Maximum vertical force	1500 N (2 - 3 times body weight)

Of the testing machines in use only the Stuttgart Athlete comes near to simulating the forces and contact times of this model, viz:

Spring stiffness	40 kN/m on contact area 140 mm diameter.
Mass	20 kg
Maximum vertical force	1500 N

The Stuttgart Athlete also offers the advantage that its outputs (force and displacement) can be used directly to determine the energy loss or dissipation during the surface contact.

The Stuttgart Athlete

It is appropriate to describe the mechanics of the Stuttgart Athlete before examining its usefulness in the measurement of impact energy loss.
The action of the instrument consists of dropping a 20 kg mass from a height of about 125 mm onto a steel spring of constant 40 N/m placed on the resilient surface. A contacting foot of 140 mm diameter and mass 3.5 kg is fixed to the bottom end of the spring.
The vertical movement of the foot caused by the impact is measured by displacement transducers and the force on it by a force transducer.

The response to the impulsive force produced by the falling mass may be modelled on that of a two mass vibration system, the motion of which can be described by the differential equations:

$$Mx'' = F(t) - K\,(X - x) - R\,(X' - x')$$

$$mx'' = -k\,x - R\,x' + K\,(X - x) + R\,(X' - x')$$

where
- M, K and R represent the drop mass (20 kg), steel spring constant (40 kN/m) and damping constant of the steel spring,
- m, k and r represent the foot mass, effective spring constant of the surface and damping constant of the surface material,

X and x are the displacements of the drop mass and the foot mass, and the primes and double primes the first and second time derivatives of these displacements, and
F(t) the impulse force applied by the drop mass.
For this analysis it is further assumed that the mass of the contact anvil is 3.5 kg, and the spring constant of the resilient surface 100 kN/m with damping constant 50.

The frequency response of the motion of the contact foot is shown in Fig. 1 from which it can be seen that it displays a resonance around 30 Hz, this being dependent on the effective spring constant of the surface and to a lessor extent the steel spring.

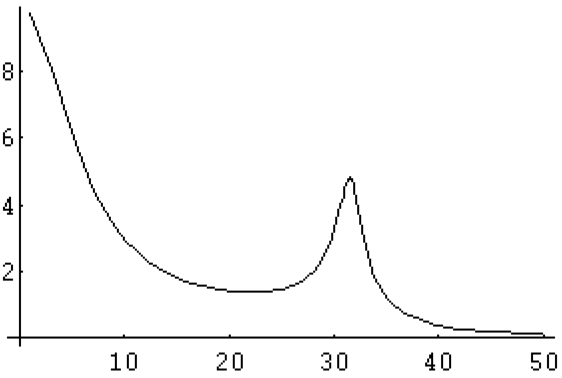

Fig. 1 Frequency (Hz) response of Stuttgart Athlete foot

The frequency response of the drop mass shows a resonance at the frequency of the 20 kg mass on a 40 kN/m spring ie. 7.1 Hz, lowered slightly by the effective spring constant of the surface.

Typical examples of the displacement and force output data from the Stuttgart Athlete are shown in Fig. 2 and 3

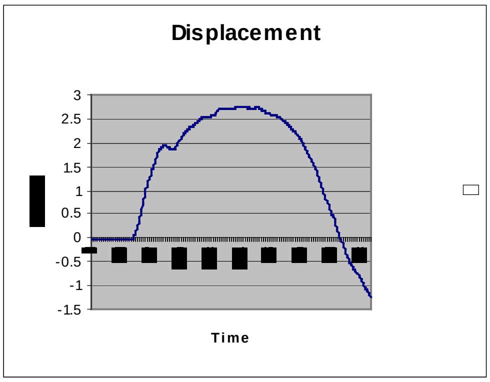

Fig 2 Displacement of foot (max = 5.6 mm)

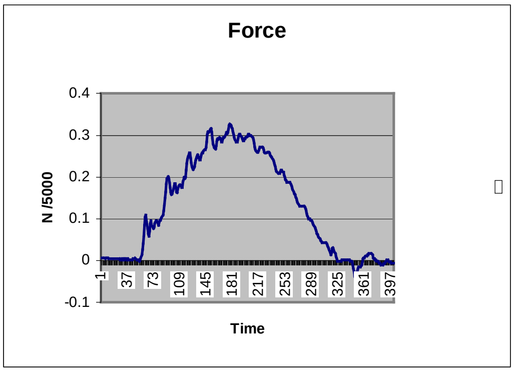

Fig. 3 Force on foot (max = 1500 N)

The force curve (Fig. 3) shows the expected half sine wave with a half time (~70 ms) corresponding to a 7 Hz resonance. There are higher frequency components seen in this force curve, which agree approximately with the frequencies of longitudinal standing wave vibrations in the spring used (k=40 kN/m, L= 50 mm, mass = 0.5 kg)
The displacement curve shows an anomaly from the expected half sine curve, most likely due to the resonance seen in the frequency response of the foot, Fig. 1.

Vertical compression of sports surfaces

The dynamic compression of carpets, which is similar to that of pile sports surfaces, has been investigated and reported by amongst others Dunlop & Sun (1989) and the results are relevant to testing for energy dissipation. The typical compression stress strain graph of a carpet is shown in Fig. 4. The loss of energy during a compression and release cycle as shown in the figure is given by the area of the loop enclosed by the compression and release curves.

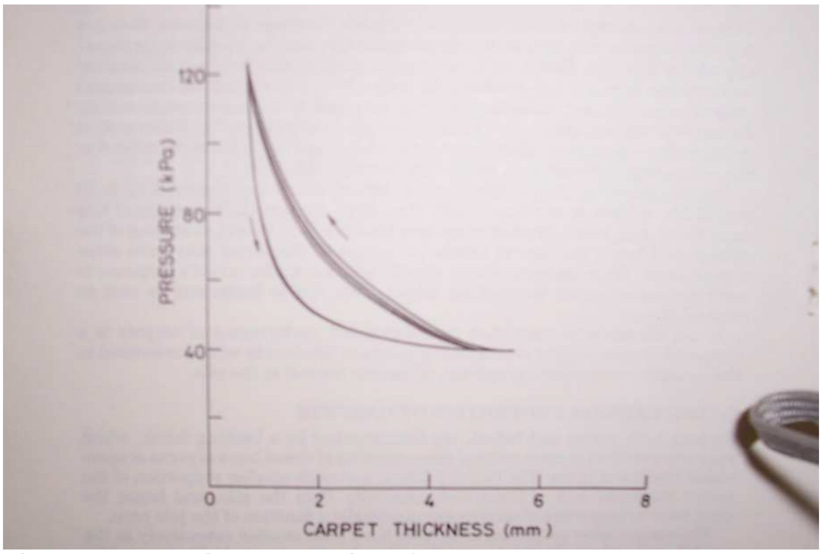

Fig. 4 Stress strain compression of carpet

Stuttgart Athlete Stress Strain

The data output from the Stuttgart Athlete shown in Fig. 3 and 4 can be replotted as a similar stress strain compression release graph as shown in Fig. 5.

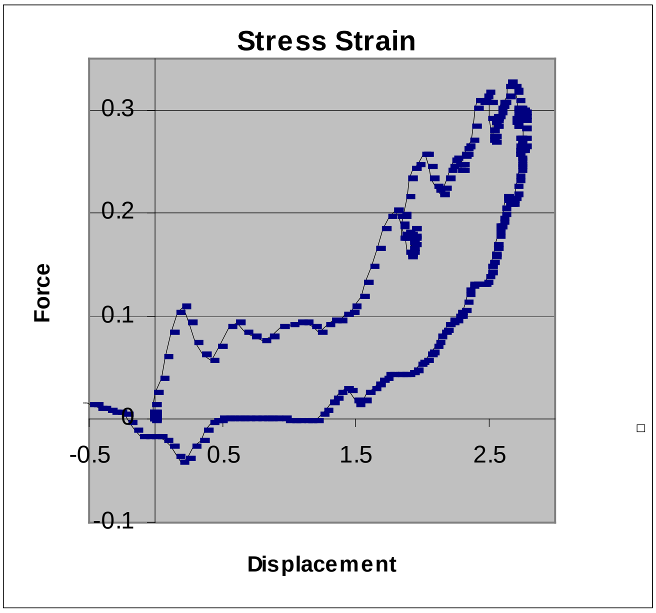

Fig. 5 Stress strain graph of Stuttgart output of force and displacement.

This graph in Fig. 5 exhibits the expected energy loss loop of a compression release cycle. However super imposed on the curves are deviations from smoothness due to the characteristics of the Stuttgart Athlete displacement and force outputs seen in Fig. 2 and 3.

The energy within the loop is the energy dissipation of the impact and can be readily obtained from the digitised data. In the example shown in Fig. 5,

Energy of compression = 432 (3.75 J)
Energy released = 96 (0.83 J)
energy loss in cycle = 336 (2.92 J)

A more convenient way of expressing the dissipation is by the parameter Energy Restitution (ER), defined as the energy returned as a percentage of the energy of compression. This parameter obviates the need for an absolute calibration of the stress and strain variables.

The Energy Restitution of the example shown (Result 85 below) is 22 %.

Field Test Results

The results of recent testing on a Football pitch (65 mm rubber/sand filled pile on a shock pad) are summarised in Table 1. These measurements were made at different positions distributed over the complete pitch.

Table 1. Results of field tests on a Football pitch (courtesy of SportsLabs Ltd.)

	Position	% ER	F max	D max
	74*	12	1503	8.4
	75	15	1503	6.5
	76	21	1503	6.1
	77	24	1457	5.3
	78	19	1548	6.0
	79	22	1503	5.4
	80	17	1412	7.6
	81	13	1457	6.5
	82	20	1457	5.9
	83	17	1457	6.3
	84	22	1366	6.6
	85*	22	1457	6.5
	86	16	1412	6.8
	87	21	1457	6.6
	88	23	1457	5.5
	89	23	1457	5.4
Mean		19.67	1463	6.19
SD		3.33	44	0.64
%		16.94	3.0	10.27

*Position 74 results rejected;
*Position 85 results Fig. 2,3 & 5

The Table shows a scatter of values for ER giving an average of 20 +/- 3 %.
The scatter of values can be attributed to two sources – the variability of the surface tested and the Uncertainty in the measurement of ER.

Uncertainty in measurement of ER

The Energy Restitution of a resilient surface is derived from the areas below the stress strain curves shown in Fig. 5. There are significant deviations from the smooth compression curves of Fig. 4 due the characteristics of this dynamic mechanical measurement mentioned earlier which must contribute to the Uncertainty in the determination.
These deviations are averaged to a large degree by assuming the full curves in the calculation. The scatter or deviations from the curves are about 20 %, but when the average is used the standard deviation of the sample used to calculate the area below the curve decreases due to the large sample size (~ 20/√50 = 2.8%).

There are other Uncertainties to consider.
The results are dependent on a peak force of 1500 N and the drop height used in testing must be adjusted to give this peak force; linear correction of the peak force to 1500 N will introduce

unknown errors as the relationship between peak force and ER is most likely non-linear. The difference from 1500 N might more conservatively be subsumed into the general Uncertainty. And the force transducer must be calibrated to traceable Standards.

Also the outputs from the force and displacement transducers must be linear over the range of operation otherwise the shape of the compression curves may be altered and error again introduced into the calculations.

A conservative estimate of the Uncertainty involved is:

Curve	2.8 %	
Force calibration	1.0 %	
Linearity displacement	0.5 %	
Linearity force	0.5 %	
Maximum force	3.0 %	(~ 50 N)

Combining these uncertainties as per the ISO *Guide to the Expression of Uncertainty in Measurement* 1993 yields the following in measurement of relative energies:

Variable	Distb	U or a	k (95 %)	df	u(I)	c(I)	u(I)c(I)	u(I)c(I)^2	u c^4/df
Curve	% Normal	2.8	1.97	200	5.52	1	1.42	2.0163	0.0203
Force cal	% Normal	1.0	2.18	12	2.18	1	0.46	0.2106	0.0037
Disp. Lin.	% Normal.	0.5	2.23	10	0.22	1	0.22	0.0504	0.0003
Force Lin.	% Normal	0.5	2.23	10	0.22	1	0.22	0.0504	0.0003
Max force	% Normal	3.0	2.13	15	1.41	1	1.41	1.9810	0.2616
				Sums				4.3087	0.2862
				combined s.d.			2.08		
				Effective df			64.87		
				k (95 %) =			2.00		
				Expanded uncertainty			4.15		
				rounded			4.15		

Total Uncertainty in determination of Energy Restitution is then:

Energy of compression (say 100 J)	4.1 %	(100 +/- 4.1)
Energy of release (say 20 J)	4.1 %	(20 +/- 0.8)
ER	8.3 %	(20 +/-1.7)

Validation

Validation of a result is the confirmation of its value using an independent method to obtain it. The Energy Restitution value determined here is the ER for a particular test method (an impact as produced by the Stuttgart Athlete) and so it should be noted that this is not a validation of the absolute value obtained or that the results are correlated to human activity on these surfaces. Hence it will be sufficient for validation purposes to use the output data from the machine and to determine ER using a different method.

An alternative method of calculating the Energy Restitution is by examining the velocity, and hence kinetic energy of the contact mass at the beginning and at the end of the impact. A half sinusoidal force increment (Fig. 3) is applied to the contact mass at impact. This causes it to accelerate to an initial velocity (V_i), which is then absorbed by the springiness of the surface and returned after impact with a reduced final velocity (V_f) due to the dissipation of energy. The ratio of final to initial kinetic energies $(V_f/V_i)^2$ yields the Energy Restitution. The velocity of the mass during contact can be obtained from the time derivative of the Stuttgart displacement and is shown for the example in Fig. 6.

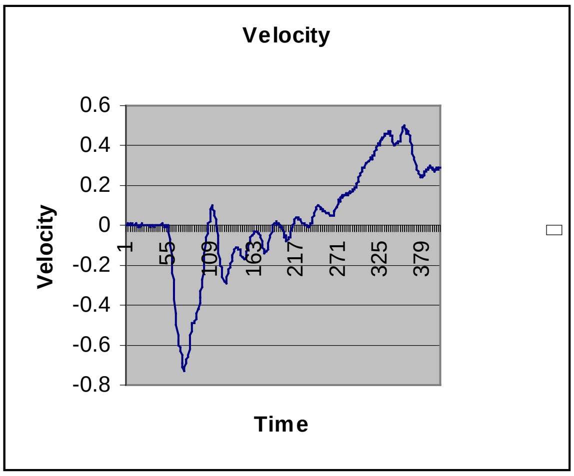

Fig. 6 Velocity of the foot during the impact cycle

A problem arises in that on initial application of the force the velocity is zero, because of the inertia of the mass. After contact the mass accelerates and reaches a maximum before being decelerated by the surface material resistance. An estimate of what the initial velocity would have been without the inertia effect may be obtained by linearly extrapolating the velocity curve back to the initial time of contact.

The Energy Restitution can then be obtained by the equation

$$ER = (V_f/V_i)^2 *100\ \%$$

In the example shown V_i = 1.0 (+/- 0.2) (following extrapolation)
V_f = 0.50 (+/- 0.01)
ER = 24 % (+/- 10)

This agrees satisfactorily with the result obtained by the loop area method.

Discussion

There is some evidence that measurement of Energy Restitution may be a useful tool in characterising the dynamic properties of sports surfaces. Here I have examined the use of the Stuttgart Athlete in achieving this aim.

The Stuttgart Athlete has several advantages for this purpose in that its actions are reasonably close to that of the mechanical model, the McMahon-Greene model, generally used to represent the actions and reactions of an athlete running on a resilient surface. And the output parameters of the apparatus are readily amenable to the determination of Energy Restitution.

However, in concordance with similar mechanical devices used to characterise surface resilience, its actions produce side effects, internal mechanical resonances, and responses different from athlete's actions, which may produce inconsistencies in the results.

Several tests have been performed on one Football pitch and the equipment has yielded results that are both consistent (ER = 20 +/- 3 %) and credible.

Bibliography

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