Uncertainty in Berlin Athlete Measurements

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The Berlin Athlete measures the mechanical resilience of a sports surface in terms of Force Reduction and is widely used for monitoring the quality of sports installations. Here we compute the uncertainty in the determined value of Force Reduction (FR) due to uncertainties in the various measurement variables. The analysis is based on the ISO publication *Guide to the Expression of Uncertainty in Measurements, 1992,* (GUM*).*

The Berlin Athlete apparatus involves dropping a 20 kg mass onto a spring, rated at 2 MN/m, and measuring the peak force developed. The peak force developed on a test sample, $F_{s,}$ is compared to the force developed on concrete, $F_{c,}$ and an expression for Force Reduction is evaluated viz.

$$FR = 100(1- F_s/F_c)$$

The uncertainty of the measurement may be calculated as per GUM which uses a spreadsheet to combine the uncertainty associated with each of the components of the measurement chain (assuming that they are uncorrelated) and yields a final uncertainty at the 95 % confidence level. A typical spreadsheet for the measurement of Force Reduction on steel spring *Reference Normals* is shown in Table 1.

Table 1. – F_s/F_c uncertainty analysis of typical Berlin Athlete data for a *Reference Normal* spring.

Component	Units	Distribution	U or a	Coverage k	df	u(I)	c(I)	u(I)c(I)
Spring	%	normal	0.76	2.31	8	0.3290043	1	0.3290043
mass	%	normal	0.25	2	30	0.125	0.5	0.0625
drop height	%	normal	0.45	2	30	0.225	0.5	0.1125
conc. resol.	%	Rect	0.05	1.732	100000	0.0288684	1	0.0288684
conc. meas.	%	normal	1	2.78	4	0.3597122	1	0.3597122
samp. resol.	%	Rect	0.05	1.732	100000	0.0288684	1	0.0288684
samp. meas.	%	normal	1	2.78	4	0.3597122	1	0.3597122

FR	Uncertainty
55	0.68
40	0.85
35	0.89
30	0.92

Sums		
combined s.d.		0.6206923
Effective df		15.081178
k =		2.12
Expanded uncertainty		1.3158676
rounded	%	1.32
Uncertainty in FR		0.92

Some explanation of the spread-sheet. The first few column headings are as follows:

Component – that element or variable in the measurement chain
Units of uncertainty - here all in %
Distribution – statistical distribution for calculating uncertainty
U or a – uncertainty of the element
Coverage factor – statistical term, derived from the t distribution and number of degrees of freedom.
df – degrees of freedom, closely tied to the number of measurements
u(i) – normalised uncertainty, or standard deviation of the mean
c(i) - weighting factor due to the calculation formula

The *combined s.d.* (standard deviation) is the square root of the sums of squares of each standard deviation shown in the column headed u(i)c(i).

The *Effective df* (df_{eff})is calculated using the Welch-Satterthwaite equation

$$df_{eff} = u_c^4 / \Sigma(u(i)^4\ c(i)^4/df)$$

where u_c is the combined standard deviation

The following typical values have been inserted into Table 1:

Spring – the uncertainty given by spring manufacturer (Rein) is 3 %. No coverage factor is given so it is assumed to result from the 11 measurements shown on their certificate. The degrees of freedom are thus 10 less some due to their reference chain to National Standards, giving say 8.
The weighting factor is calculated from the relationship between the Force Reduction value and the spring constant. The measured force, F, is related to the square root of the spring rate k via:

$$F^2 - 2mgF = 2mghk \quad \ldots. \qquad \text{equation 1}$$

thus giving a weighting factor of approximately a half.

However, Force Reduction of the sample is related to F_s/F_c, where F_s is due to the effective spring constant, the combination of the Berlin Athlete spring and the sample spring constant, here assumed to be a simple series combination. The relationship of FR to the spring constant is therefore not a simple one.

To simplify the analysis of the uncertainty of F_s/F_c the following approximation of equation 1 is made:

$$F^2 = 2\ m\ g\ h\ k$$

This approximation results in an error in FR of 1.1 for FR = 35 %, or 3 % so it is justified here. This then gives

$$F_s/F_c = \oplus\ (k_s / k_0)$$

Where k_s is the effective spring constant of the system.
Then the uncertainty in F_s/F_c ($🙤_s$) due to the uncertainty in Berlin Athlete spring constant ($🙤_k$) is derived as

$$🙤_s = ½\ k_0/(k + k_0\ )\ 🙤_k$$

where k and k_0 are the spring constants of the sample and of the Berlin Athlete spring
This yields uncertainties in F_s/F_c at the following Force Reduction values, due to the spring uncertainty of 3%:

FR = 55 %, $🙤_s$ = 1.12 ;
FR = 40 %, $🙤_s$ = 0.96 ;
FR = 35 %, $🙤_s$ = 0.86 ;
FR = 30 %, $🙤_s$ = 0.76.

with a weighting factor of unity.

These values have been used in Tables 1 and 2.

Mass – tolerance from DIN 18032 is 0.25 %. Assuming many readings the coverage factor = 2. But the parameter F_s/F_c is independent of mass m for a linear spring system. However the system is expected to be non-linear so a weak dependence on m for the uncertainty in $🙤_s$ is assumed, a weighting factor of 0.5 being used.

Drop height – tolerance from DIN 18032 is 0.45 %. Assuming many readings so coverage factor = 2. Weighting factor as for mass m.

Resolution – assuming 12 bit recording, and measurements at half full scale gives resolution of 0.05 % top and bottom, or 0.1 %. This is a rectangular distribution so coverage factor is 1.732 for 95% level.

Measurements of F – Round Robin testing uses steel springs which are assumed elastic so there is no limit on the number of impacts – five are specified in the ISSS certification rounds.
Sports surfaces however are generally inelastic and repetitive impacts may change the characteristics of the surface. This is particularly noticeable with synthetic turf, filled and unfilled, and testing protocols, eg. FIH, specify an initial drop and then taking the average of the next two impacts on the same spot. For athletic track surface, solid or layered polymeric, the differences between drops are not so apparent but the IAAF protocol specifies the same measurement protocol.

The uncertainty in the final result as a consequence of this small number of readings is great because of the large value of the coverage factor – it is 12.7. An analysis of the uncertainty is shown in Table 2.

For these sports surfaces it is suggested that a preliminary drop be made and the next two impacts measured, and the process then repeated on an adjacent spot, an average being taken of the four measurements. This will probably result in a larger standard deviation than if the measurements were made on the same spot, due to variations in sample properties.

A typical uncertainty (Experimental Uncertainty of the Mean) of 1.0 % (Table 1) has been observed for 5 measurements on the *Reference Normal* spring - this corresponds to a standard deviation of 0.8 %, from archival data.
The uncertainty for the two measurements on the sports surface might then be 12.7 % (Table 2) assuming a standard deviation of 1.0 %, slightly more than the 0.8 % for the spring due to machine siting on a rougher surface.
The uncertainty for the four measurements at two positions on the sports surface might be 2.5 % (Table 3) assuming a standard deviation of double that for the spring or 1.6 %.

The calculations shown in Table 1 indicate the final uncertainty in the ratio F_s/F_c for a *Reference Normal* spring of FR in the range 30 to 55 %.

The uncertainty in FR is the absolute uncertainty value of this F_s/F_c percentage uncertainty. Calculated uncertainties in FR are shown (lower left in the Tables) at the following FR values – 55 % (the FIFA limit), 40 % (the FIH limit), 35 % (the IAAF limit) and 30 %.

For measurements on a sports surface the uncertainty is greater due to reasons discussed above and is shown in Table 2.

The uncertainty using the modified testing protocol of 4 readings is shown in Table 3.

Table 2. – F_s/F_c uncertainty analysis of typical Berlin Athlete data for a sports surface.

Component	Units	Distribution	U or a	Coverage k	df	u(I)	c(I)	u(I)c(I)
Spring	%	normal	0.76	2.31	8	0.3290043	1	0.3290043
mass	%	normal	0.25	2	30	0.125	0.5	0.0625
drop height	%	normal	0.45	2	30	0.225	0.5	0.1125
conc. resol.	%	Rect	0.05	1.732	100000	0.0288684	1	0.0288684
conc. meas.	%	normal	1	2.78	4	0.3597122	1	0.3597122
samp. resol.	%	Rect	0.05	1.732	100000	0.0288684	1	0.0288684
samp. meas.	%	normal	12.7	12.7	1	1	1	1

FR	Uncertainty
55	3.53
40	3.93
35	4.15
30	4.51

Sums		
combined s.d.		1.1206543
Effective df		1.5683288
k =		6.7
Expanded uncertainty		7.5083836
rounded	%	7.51
Uncertainty in FR		4.51

Table 3. – F_s/F_c uncertainty analysis of typical Berlin Athlete data for a sports surface using four readings.

Component	Units	Distribution	U or a	Coverage k	df	u(I)	c(I)	u(I)c(I)
Spring	%	normal	0.96	2.31	8	0.4155844	1	0.4155844
mass	%	normal	0.25	2	30	0.125	0.5	0.0625
drop height	%	normal	0.45	2	30	0.225	0.5	0.1125
conc. resol.	%	Rect	0.05	1.732	100000	0.0288684	1	0.0288684
conc. meas.	%	normal	1	2.78	4	0.3597122	1	0.3597122
samp. resol.	%	Rect	0.05	1.732	100000	0.0288684	1	0.0288684
samp. meas.	%	normal	2.5	3.18	3	0.7861635	1	0.7861635

FR	Uncertainty
55	1.06
40	1.39
35	1.51
30	1.65

Sums	
combined s.d.	0.9687031
Effective df	6.5106682
k =	2.4
Expanded uncertainty	2.3248874
rounded %	2.32
Uncertainty in FR	1.39

Further Uncertainties

The frequency response of the Force Transducer and amplifier may influence the results obtained using the Berlin Athlete and the effects of variations between machines using the same nominal filter and different transducers are difficult to estimate.

Generally, the amplifier response might correspond to Channel Class specifications eg. ISO 6487 1987 which nominates 0.1 Hz or lower cut-off at the bottom end, flat (+/- 0.5 dB) to a specified upper cut-off frequency, with four pole attenuation.

Under the present regime the filter frequency response at the upper end is usually calibrated but there is no other requirement than a static calibration of the force transducer. The complete frequency response of the total system (transducer and amplifier) should be calibrated, as it is with accelerometers for *Critical Fall Height* Standards, eg ISO 6487.

Mechanical peculiarities between machines, for example different friction losses and indeterminate amounts of energy lost at the impact, will also produce uncertainty in the result.

Round Robin (validity) tests can reduce the uncertainty between machines, and may be effective in reducing some of these further uncertainties, but do not contribute greatly to reducing the uncertainty of the absolute measurement.

These various effects then add another layer of uncertainty (25 % conservatively) to our model, giving, for example, an uncertainty of 1.9 (1.51 plus 25 %) in a laboratory measurement (four readings) of track material where FR = 35 %. Using current testing protocols the uncertainty is 5.1.

Conclusion

An analysis has been made of the uncertainty of measurements using the Berlin Athlete specified in DIN 18032, based on the ISO publication *Guide to the Expression of Uncertainty in Measurements, 1992.*

The expected uncertainty in the determined value of Force Reduction for the laboratory testing of sports surface samples, by the testing protocol suggested here (four readings), would probably be not less than

1.3 at FR = 55 %, the FIFA limit,
1.7 at FR = 40 %, the FIH limit and
1.9 at FR = 35 %, the IAAF limit.

Testing in the field is generally less accurate than laboratory testing due to the environment (wind, rain, glare, heat, humidity, dust and slope) so these stated uncertainties in the determined value of Force Reduction might be increased by up to 25 % depending on conditions.

To current practitioners these uncertainty estimates may appear to be high. However they should be considered in relation to traditional geo-mechanical methods for measuring the elastic modulus of surfaces, where uncertainties of 10 to 50 % (relative) are often acceptable.

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